Домашнее задание

1.1

1. $x^4 + 2x^2 - 8 = 0$ — $x = \pm\sqrt{2}$
2. $x^4 - 13x^2 + 36 = 0$ — $\{\pm 3,\ \pm 2\}$
3. $x^6 - 3x^3 + 2 = 0$ — $\{1,\ \sqrt[3]{2}\}$
4. $2x^8 + x^4 - 15 = 0$ — $x = \pm\sqrt[4]{\frac{5}{2}}$
5. $3x^2(x-1)(x+1) - 10x^2 + 4 = 0$ — $\left\{\pm 2,\ \pm\frac{1}{\sqrt{3}}\right\}$

1.2

6. $(x^2 - 2x)^2 - (x-1)^2 + 1 = 0$ — $\{0,\ 2,\ 1 \pm \sqrt{2}\}$
7. $(x^2 + 5x)^2 - 2(x^2 + 5x) - 24 = 0$ — $\{-6,\ -4,\ -1,\ 1\}$
8. $x^4 - 2x^3 + x - 12 = 0$ — $x = \frac{1 \pm \sqrt{17}}{2}$
9. $(x-2)^6 - 19(x-2)^3 = 216$ — $\{0,\ 5\}$
10. $(x^2 + x + 1)(x^2 + x + 2) = 12$ — $\{-2,\ 1\}$
11. $(x^2 - 5x + 7)^2 - (x-2)(x-3) = 1$ — $\{3,\ 2\}$
12. $(x^2 + x + 1)(2x^2 + 2x + 3) = 3(1 - x - x^2)$ — $\{-1,\ 0\}$
13. $(2x^2 + 3x - 2)(5 - 6x - 4x^2) = -5(2x^2 + 3x + 2)$ — $\left\{-\frac{3}{2},\ 0,\ \frac{-3 \pm \sqrt{65}}{4}\right\}$
14. $(x-2)(x+1)(x+4)(x+7) = 19$ — $\left\{\frac{-5 \pm \sqrt{85}}{2},\ \frac{-5 \pm \sqrt{5}}{2}\right\}$
15. $(6x+5)^2(3x+2)(x+1) = 35$ — $x = \frac{-5 \pm \sqrt{21}}{6}$
16. $(x^2 - 9x + 20)(x^2 - 13x + 42) = 1680$ — $\{-1,\ 12\}$
17. $x^4 + 15x^2 + 2x^3 + 14x + 24 = 0$ — $\varnothing$

1.3

18. $x^4 + (x-4)^4 = 82$ — $\{1,\ 3\}$?
19. $(x+3)^4 + (x+5)^4 = 16$ — $\{-3,\ -5\}$?
20. $(x-1)^5 + (x+3)^5 = 242(x+1)$ — $\{-2,\ -1,\ 0\}$?
21. $(1+x)^8 + (1+x^2)^4 = 4x^4$ — $\varnothing$
22. $10x^2(x-2)^2 = 9x^2 + 9(x-2)^2$ — $\{-1,\ 3\}$?

1.4

20. $x^4+x^3+x+1=18x^2$ $\left\{2\pm\sqrt{3},\ \frac{-5\pm\sqrt{21}}{2}\right\}$?

21. $x^4-6x^3+10x^2-6x+1=0$

22. $x^4-7x^3+14x^2-7x+1=0$ $\left\{2\pm\sqrt{3},\ \frac{3\pm\sqrt{5}}{2}\right\}$

23. $x^4+5x^3+2x^2+5x+1=0$ $x=\frac{-5\pm\sqrt{21}}{2}$

24. $2x^4+3x^3-4x^2-3x+2=0$ $\left\{-2,\ \pm1,\ \frac{1}{2}\right\}$

25. $x^4+1=2(x+1)^4$ $x=\frac{-4+\sqrt{6}\pm\sqrt{18\pm8\sqrt{6}}}{2}$

26. $x^6-9x^5+26x^4-33x^3+26x^2-9x+1=0$ $\left\{\frac{3\pm\sqrt{5}}{2},\ \frac{-5\pm\sqrt{21}}{2}\right\}$

27. $x^4-2x^3-18x^2-6x+9=0$ $\left\{-1,-3,\ 3\pm\sqrt{6}\right\}$?

28. $4x^4-8x^3+3x^2-8x+4=0$ $\left\{2,\ \frac{1}{2}\right\}$?

1.5

29. $\left(x^2+27\right)^2-5\left(x^2+27\right)\left(x^2+3\right)+6\left(x^2+3\right)^2=0$ $\left\{\pm3,\ \pm\sqrt{21}\right\}$

30. $\left(x^2-3\right)^2-7\left(x^4-9\right)+6\left(x^2+3\right)^2=0$ $\varnothing$

31. $\left(x^2+2x+2\right)^2+x\left(x^2+2x+2\right)=30x^2$ $\left\{1,\ 2,\ -4\pm\sqrt{14}\right\}$

32. $(x-2)^2(x+1)^2-(x-2)\left(x^2-1\right)-2(x-1)^2=0$ $\left\{0,\ \pm\sqrt{3},\ 3\right\}$

33. $\left(x^2+27\right)^2-5\left(x^4+30x^2+81\right)+6\left(x^2+3\right)^2=0$ $\left\{\pm3,\ \pm\sqrt{21}\right\}$

34. $2\left(x^2+x+1\right)^2-7(x-1)^2=13\left(x^3-1\right)$ $\left\{-1,\ -\frac{1}{2},\ 2,\ 4\right\}$

35. $\left(x^2+4x+8\right)^2+3x^3+14x^2+24x=0$ $\{-4,\ -2\}$?

36. $(x+2)(x+3)(x+8)(x+12)=4x^2$ $\left\{-6,\ -4,\ \frac{-15\pm\sqrt{129}}{2}\right\}$

37. $-4x\left(x^2+8\right)+\left(x^2-x+8\right)^2=73x^2$

38. $\left(x^2-6x-9\right)^2=x\left(x^2-4x-9\right)$ $\left\{-1,\ 9,\ \frac{5\pm\sqrt{61}}{2}\right\}$?

39. $\left(x^2-2\right)\left(x^2+3x-5\right)=4(x-1)^2$ $\left\{\frac{1\pm\sqrt{5}}{2},\ -2\pm\sqrt{10}\right\}$

40. $\left(2x^2+x\right)^2+x^2=12(2x+1)^2$ $x=\frac{3\pm\sqrt{15}}{2}$

41. $(2x+3)(2-2x)(4x^2+12x+14)=4(2x+4)^2$ $\left\{\frac{-1\pm\sqrt{3}}{2}\right\}$?

42. $(4x+1)(5-4x)(16x^2+8x+7)=5(4x+2)^2$

1.7

43. $x^3-(\sqrt{2}+1)x^2+2=0$ $\left\{\sqrt{2},\ \frac{1\pm\sqrt{4\sqrt{2}+1}}{2}\right\}$

44. $x^4-2\sqrt{3}x^2+x+3-\sqrt{3}=0$

45. $x^4+x^2(1-2\sqrt{5})-2x-2\sqrt{5}+5=0$ $\left\{\frac{-1\pm\sqrt{4\sqrt{5}-7}}{2},\ \frac{1\pm\sqrt{4\sqrt{5}+1}}{2}\right\}$

2.1

46. $(4+x)^2=(4+x)(17x+2)$

47. $3x^6+12x^4-96x^2=0$ $\{0,\pm2\}$

48. $x^5+5x^3-6x^2=0$ $\{0,1\}$

49. $x^4-2x^3+2x-1=0$ $x=\pm1$

50. $2x^3+x^2-4x-2=0$ $\left\{-\frac{1}{2},\pm\sqrt{2}\right\}$

51. $x^2-4=x^3+8$ $x=-2$

52. $(x-1)^3+(2x+3)^3=27x^3+8$ $\left\{-\frac{2}{3},\ -\frac{1}{2},\ 3\right\}$

53. $8x^3-36x^2+54x=28$ $x=2$

54. $16x^4+32x^3+24x^2+8x-80=0$ $\{-2,\ 1\}$

55. $x^4-8x^3+24x^2-32x=65$ $\{-1,\ 5\}$

56. $5x^3-6x^2-6x-2=0$ $x=\frac{\sqrt[3]{2}}{\sqrt[3]{7}-\sqrt[3]{2}}$

57. $(x^2+4x+3)^2+(x^2-2x-15)^2=36(x+3)^2$ $\{-3,\ -1,\ 5\}$

58. $\frac{(x^2+25x+24)^2}{3}+\frac{3\left(2x^2-\frac{14}{3}x-\frac{20}{3}\right)^2}{4}=\frac{(x^2+27x+26)^2}{3}$ $\left\{-1,\ 0,\ \frac{4}{3}\right\}$

59. $(x^2+4x+3)^2+(x^2+3x+2)^2=(x^2-1)^2+(x^2-x-2)^2$ $\left\{-1,\ -\frac{1}{2}\right\}$?

60. $x^4-4x-1=0$ $x=\frac{\sqrt{2}\pm\sqrt{4\sqrt{2}-2}}{2}$

61. $x^4-4x^3-1=0$ $x=\frac{\sqrt{2}\pm\sqrt{4\sqrt{2}-2}}{2(\sqrt{2}-1)}$

2.2

62. $x^3-6x^2+15x-14=0$ $\quad x=2$

63. $6x^3-x^2-20x+12=0$ $\quad \left\{-2,\ \frac{3}{2},\ \frac{2}{3}\right\}$

64. $x^3+4x^2+6x+3=0$ $\quad x=-1$

65. $x^3+9x^2+23x+15=0$ $\quad \{-5,-3,-1\}$

66. $2x^3-x^2-5x-2=0$ $\quad \left\{-1,\ -\frac{1}{2},\ 2\right\}$

67. $x^3+6x^2-x-30=0$

68. $(x+1)^2(x+2)+(x-1)^2(x-2)=12$ $\quad x=1$

69. $2x^4-x^3+2x^2+3x-2=0$ $\quad \left\{-1,\ \frac{1}{2}\right\}$

70. $x^4-9x^3+30x^2-44x+24=0$ $\quad \{2,\ 3\}$

71. $3x^4+5x^3-x^2-5x-2=0$ $\quad \left\{\pm 1,\ -\frac{2}{3}\right\}$

72. $8x^4+6x^3-13x^2-x+3=0$ $\quad \left\{-\frac{1}{2},\ \frac{3}{4},\ \frac{-1\pm\sqrt{5}}{2}\right\}$

73. $2x^4-x^3-9x^2+13x-5=0$ $\quad \left\{-\frac{5}{2},\ 1\right\}$

74. $4x^4-16x^3+3x^2+4x-1=0$

75. $x^5+5x^4+3x^3-13x^2-8x+12=0$ $\quad \{-3,\ -2,\ 1\}$

76. $2x^5+5x^4-13x^3-13x^2+5x+2=0$

77. $\left(x^2-3x+1\right)\left(x^2+3x+2\right)\left(x^2-9x+20\right)=-30$ $\quad x=\frac{3\pm\sqrt{25\pm4\sqrt{30}}}{2},\ x=\frac{3\pm\sqrt{29}}{2}$

2.3

78. $\left(x^2+2x-5\right)^2+2\left(x^2+2x-5\right)-5=x$ $\quad \left\{\frac{-1\pm\sqrt{21}}{2},\ \frac{-3\pm\sqrt{17}}{2}\right\}$

79. $\left(x^2-x-3\right)^2-\left(x^2-x-3\right)-3=x$ $\quad \left\{-1,\ 3,\ \pm\sqrt{3}\right\}$

3.

80. $\left(x^2+8x+7\right)^3=(2x-1)^3$

81. $\left(x^2+3x-23\right)^3=(4x-3)^3$

82. $\left(x^2-11x+9\right)^2=(2x+9)^2$

83. $\left(x^2-12x+10\right)^4=(3x+10)^4$

84. $3\left(x^2-4x-12\right)^2=4x^4$

85. $\left(4x^2+3x-10\right)^2=9x^4$

86. $169x^4=\left(x^3+40x\right)^2$

87. $144x^4=\left(x^3+35x\right)^2$

88. $\dfrac{\left(x^2-\dfrac{1}{32}x-\dfrac{1}{96}\right)^3}{8}=\dfrac{\left(x^2-\dfrac{3}{64}x+\dfrac{1}{64}\right)^3}{27}$

89. $\dfrac{\left(x^2+\dfrac{5}{32}x-\dfrac{1}{32}\right)^3}{27}=\dfrac{\left(x^2+\dfrac{5}{24}x+\dfrac{1}{24}\right)^3}{64}$

90. $(12x-49)^{25}+(2x+7)^{50}=0$